Name: Solutions

Date: 2/24/2025

Math 130

Quiz 6

Some formulas you may need:

$P(A \cup B) = P(A) + P(B)$ $\quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $\quad P(\bar{A}) = 1 - P(A)$

$P(at\ least\ one) = 1 - P(none)$ $\quad P(A \cap B) = P(A) \cdot P(B)$ $\quad P(A \cap B) = P(A) \cdot P(B \mid A)$

1. (1, 2, 2, 2, 2, 1 points) Suppose you are going to Las Vegas for a vacation over the weekend and are trying to plan where you will be going to dinner. After doing some research, you narrowed down your dinner choices to the restaurants categorized in the table below, and you will be equally happy going to any of them.

American Cuisine	Asian Cuisine	Mexican Cuisine	Steak House	Total
2	5	4	9	20

a) If you randomly select a restaurant to go to for Friday night's dinner, what is the probability that you will not go to a steak house?

Let SH be the event that a steak house is chosen for dinner.

$$P(\overline{SH}) = 1 - P(SH) = 1 - \frac{|SH|}{|S|}$$

$$= 1 - \frac{9}{20} = \boxed{\frac{11}{20} = 55\%}$$

b) If you randomly select the restaurants to go to for both Friday and Saturday night's dinners with replacement (i.e. you are willing to go to the same restaurant on both nights), what is the probability that you'll have Asian cuisine on Friday night and you'll go to a steak house on Saturday night?

Let AC be the event that an Asian Cuisine restaurant is chosen for dinner.

independent ✓

$$P(AC_1 \cap SH_2) = P(AC_1) \cdot P(SH_2)$$

$$= \frac{5}{20} \cdot \frac{9}{20} = \boxed{\frac{45}{400} = \frac{9}{80} = 11.25\%}$$

c) If you randomly select the restaurants to go to for both Friday and Saturday night's dinners without replacement (i.e. you insisting on going to different restaurants each night), what is the probability that you'll have Asian cuisine on Friday night and you'll go to a steak house on Saturday night?

independent ?

$$P(AC_1 \cap SH_2) = P(AC_1) \cdot P(SH_2 \mid AC_1)$$

$$= \frac{5}{20} \cdot \frac{9}{19} = \boxed{\frac{45}{380} = \frac{9}{76} = 11.84\%}$$

d) If you randomly select the restaurants to go to for both Friday and Saturday night's dinners <u>without replacement</u> (i.e. you insisting on going to different restaurants each night), what is the probability that you'll go to a steak house on both nights?

indep. ?

$$P(SH_1 \cap SH_2) = P(SH_1) \cdot P(SH_2 \mid SH_1)$$

$$= \frac{9}{20} \cdot \frac{8}{19} = \boxed{\frac{72}{380} = \frac{18}{95} = 18.95\,\%}$$

e) If you randomly select the restaurants to go to for both Friday and Saturday night's dinners <u>without replacement</u> (i.e. you insisting on going to different restaurants each night), what is the probability you will not go to a steak house on either of the nights?

indep. ?

$$P(\overline{SH_1} \cap \overline{SH_2}) = P(\overline{SH_1}) \cdot P(\overline{SH_2} \mid \overline{SH_1})$$

$$= \frac{11}{20} \cdot \frac{10}{19} = \boxed{\frac{110}{380} = \frac{11}{38} = 28.95\,\%}$$

f) If you randomly select the restaurants to go to for both Friday and Saturday night's dinners <u>without replacement</u> (i.e. you insisting on going to different restaurants each night), what is the probability you will go to a steak house on at least one of the nights?

$$P(\text{at least one Steak house}) = 1 - P(\text{no steakhouse}) = 1 - P(\overline{SH_1} \cap \overline{SH_2}) = 1 - \frac{110}{380} = \boxed{\frac{270}{380} = \frac{27}{38} = 71.05\,\%}$$

<u>Extra Credit (5 points)</u>: Suppose you decide to stay in Las Vegas for 4 nights. If you randomly select the restaurants to go to for each of the 4 night's dinners <u>without replacement</u> (i.e. you insisting on going to different restaurants each night), what is the probability that you will have Mexican cuisine on the first 2 nights and you won't have Mexican cuisine on the last 2 nights?

Let M be the event that you will have Mexican cuisine on a given night.

indep. ?

$$P(M_1 \cap M_2 \cap \overline{M_3} \cap \overline{M_3}) = P(M_1) \cdot P(M_2 \mid M_1) \cdot P(\overline{M_3} \mid M_1 \cap M_2) \cdot P(\overline{M_4} \mid M_1 \cap M_2 \cap \overline{M_3})$$

$$= \frac{4}{20} \cdot \frac{3}{19} \cdot \frac{16}{18} \cdot \frac{15}{17}$$

$$= \boxed{\frac{2880}{116{,}280} = \frac{8}{323} = 2.48\,\%}$$